# Firebrands Induced ROS Estimation

### Trouve (3/13/2022)

We consider a stationary line fire that emits embers to flight distances (measured from the fireline location). The distribution of possible values of ember flight distances is characterized statistically by the PDF of , , and is assumed known. We also assume that the PDF takes non-zero values for .

We further assume a spatial resolution and discretize the PDF of into bins as follows:

where . We can calculate as:

We have by construction:

Let us now consider that the line fire emits embers at an average rate, noted , during a duration, noted . The total number of embers emitted during that time can be expressed as:

These embers land on the ground at different values of the flight distances . Let us call the total number of embers that belong to the bin . We can write:

and

Because is a decreasing finction of , we see that the smallest number is achieved for and in order to guarantee that , one must satisfy the following condition for the ember generation rate:

This condition corresponds to a requirement for converged statistics. In the following, we call this critical value of required for converged statistics.

Note that when , we have:

These expressions correspond to the total number of embers emitted during . During a time step , the total numbers of embers emitted is:

and the total number of embers that belong to the bin is:

We now turn to an estimate of the rate of spread of the fire, noted , in the presence of embers. We assume that the ember generation rate is . Under those conditions, during the time , and in an average sense, we expect 1 ember to fly to the maximim distance . Assuming also a probability of ignition equal to 1, this leads to the following estimate of :

### Yiren

Starting from the view that the rate of spread (ROS) of fire is defined based on the period when the cell/bin including receives at least 1 ember, we may extend the definition to an instant ROS, which is the displacement of the cell/bin which receives 1 ember in unit time, i.e.,

where is the location of the cell that receives 1 ember at time . The “receiving 1 ember” criterion is defined stochastically. It means that at the ember generating rate , according to a predefined probabilistic distribution for the ember travel distance, the product gives a result of 1. is the expected total number of embers received by cell/bin during time .

has the unit per cell per second,

is a discretized probability density function, whose bin size is the same as the mesh resolution during simulation, is the mesh resolution, is the time from the start of the simulation, and is the original probability density function.

Therefore, if the profile of versus time is given, a mathematically defined ROS accounting for the fire spreading due to firebrands showering can be derived by taking 1st order derivative of , as illustrated by equation (1).

## 1 Infinite fast ember

It is noticed that will depend on the form of . As a semi-physical representation, induced by a single cell to any downwind location (bin ) can be expressed by equation (3):

It means that cells start to shoot embers when ignited at the time , and will keep emitting for a period of . Assumptions made here are (1) the ember reaches the target immediately after being emitted, (2)100% ignition, and (3) no ignition time. It is visualized in Figure 1.



Figure 1 Temporal profile of expected number of embers

One way to find is to draw the profile of versus time. The plots below give the contours for on an x-t plane. The contour line which represents is the wanted profile. Then the can be calculated by numerical differentiation.

The results shown in Figure 2 are from a truncated lognormal distribution, the Sardoy’s distribution, for wind speed 15 mph, fireline intensity 3.19 kW/m, , and .



Figure 2 Contour for on an x-t plane

Besides utilizing the capability of numerical methods, the interface can also be solved analytically. Firstly, from the expression for , we can solve the location where at time :

Therefore,

Specifically, for the Sardoy’s lognormal distribution within a truncated region, :

where

is a normalization factor which ensures . As a result, can be solved from the expression above by equation (2):

To solve for , condition must be satisfied.

Therefore, when ,

Or when

Since it is hard to solve explicitly from the equation above, solving for is done taking 1st order derivative for both sides of the above equations regarding , which is presented in the final pages. As a result:

It is noticed that when , will linearly increase with , which will lead to an infinite large when is high enough.

If assuming the at the , which can be expressed as

is the speed that dominates the general spreading of the fire, the effects of and to the are shown in Figure 3. It is noticed also increases with and converges to infinite large when is big enough.



Figure 3 Dependency of on and

Comparisons are made between the analytical and simulated results. The simulations were done by the 1-D version ELMFIRE in MATLAB. The first cell of the simulated region is able to emit embers while the others are only capable of propagating the fire through surface fire models.

Figure 4(a) is drawn by the time of arrival for each of the cells for a single simulation. The time of arrival is defined as the moment when the cell is ignited (the first timestep when ). The oscillations in the profiles are due to spot-fire ignition. Cells with a small time of arrival (low *t*-value in the figure) are ignited by the embers while the high *t*-values are fires propagating by surface fire mechanisms from the nearest ignition points. The figure shows that the analytical results partially explained that the spreading rate increases with the ember generating rate.



Figure 4 Location vs. Time of arrival under different *,*

The trends shown in the simulated results when changing other parameters such as and can also be seen in the analytical curves as is shown in Figure 5.

 

(a) (b)

Figure 5 Location vs. Time of arrival under different (a) and (b)

If using the to compare with the from simulation, as is shown in Figure 6, the differences are about an order of magnitude, although it captured the feature that when increases, the increases. Note that data in the left plots are simulated in the condition that all the cells are able to shoot embers, while plots discussed in the previous passages are for the single-cell-emitting cases.

 

Figure 6 Left: Simulated ROS profile against GR; Right: R profile against GR.

## 2 Ember traveling at wind speed

The above calculation assumes an infinite fast transport of embers. For long-range embers, assuming the embers travel at the wind speed, and the lofting/dropping time is neglectable, the number of embers landed on cell at time can be expressed as:

Still, this expression assumes only 1 cell is emitting embers. The only difference is the term , which represents the delay of ignition due to ember transport.

Similarly, we can obtain the location where as has been done for the infinite-fast-ember case, and solve for the incident :

In this condition, the maximum ROS converges to  when GR and are large enough.



Figure 7 Dependency of on and when embers traveling at the wind speed

Similar comparisons to the simulation results are made as it is done for the infinite fast ember cases.



1. (b) (c)

Figure 8 Location vs. Time of arrival under different (a), (b) , and (c)

 

Figure 9 Left: Simulated ROS profile against GR; Right: R profile against GR. Ember travels at wind speed

## ROS profile derivation

***Infinite fast ember:***

: (for simplicity, let )

In conclusion:

***Ember travels at the wind speed:***

:

In conclusion: